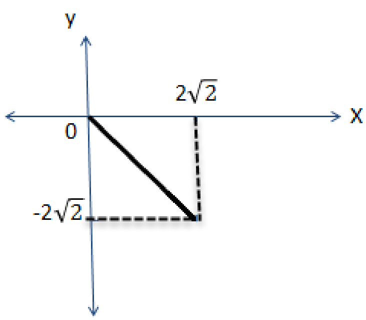
نموذج استرشادى (١) لامتحان شهادة إتمام الدراسة الثانوية العامة ٢٠٢٤/٢٠٢٥ م

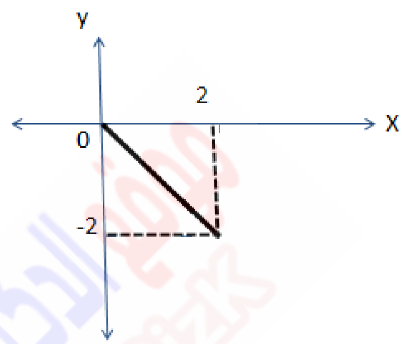
المادة: الرياضيات البحتة باللغة الإنجليزية الشعبة العلمية ــ رياضيات الزمن: ساعتان

First: Multiple choice questions" one mark for each item"

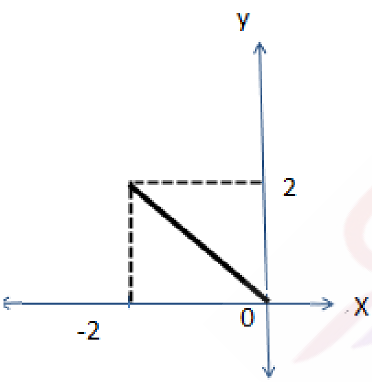
(1) Which of the following is the graphical representation of the complex number "$Z=2\sqrt{2}\left(\cos\left(\frac{-\pi}{4}\right)+i\sin\left(\frac{-\pi}{4}\right)\right)$"?



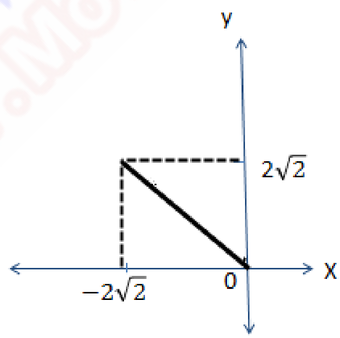
(a)



(b)



(c)



(d)

(2) The fifth term from end in the expansion $\left(x^2-\frac{1}{x}\right)^{20}$ according to descending power of x is....................

(a) $-\ {}^{20}C_4\ x^{-8}$ (b) ${}^{20}C_4\ x^{-8}$

(c) $-\ {}^{20}C_4\ x^{28}$ (d) ${}^{20}C_4\ x^{28}$

(3) If the distance between the point (a , 2a ,3a) and x y plane is 6 units length ,

then a = ... such that a > 0.

(a) 1 (b) 2 (c)3 (d) 6

(4) If $y = n^3 - 1$, $z = 1 - n^2$, then $\frac{dy}{dz} =$ .... where $n \neq 0$

(a) $-\frac{3}{2}$ (b) $-\frac{3}{2}n$ (c) $-\frac{3}{2n}$ (d) $-\frac{3}{2n^2}$

(5) If f(x) = e^{8x-x^2} , then the function is increasing in the interval ....

(a) $]-\infty, 0[$ (b) $]0, \infty[$

(c) $]4, \infty[$ (d) $]-\infty, 4[$

(6) The measure of direction angle of the vector $\vec{a}$ =($-2\sqrt{2}$, 3 , 1) with positive x-axis

$\cong$ ....

(a) $131^\circ\ 49'$ (b) $48^\circ\ 11'$ (c) 45° (d) 135°

(7) In the expansion $\left(7x + \frac{6}{x^2}\right)^{12}$ according to descending power of x ,

the term free of x is ....

(a) T_5 (b) T_6 (c) T_7 (d) T_8

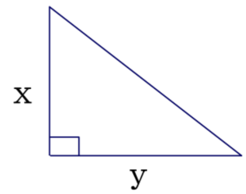
(8) The slope of the tangent to the curve y = $e^x + x$ at point (0,1) equals ....

(a) zero (b)1 (c) 2 (d) 3

(9) In the opposite figure :

If $x + y = 10$ cm ,

then the greatest area of the triangle is equal to ....cm^2



(a) 10 (b) 12.5 (c)25 (d) 100

(10) $\int \frac{7x^2}{5-4x^3}\ dx =$..................

(a) $\frac{7}{12}\ln|4x^3 - 5| + c$

(b) $\frac{12}{7}\ln|4x^3 - 5| + c$

(c) $\frac{-7}{12}\ln|4x^3 - 5| + c$

(d) $\frac{-12}{7}\ln|4x^3 - 5| + c$

<u>Second : Multiple choice questions" two marks for each item"</u>

(11) If the amplitude of $\left(\frac{z_1}{z_2}\right) = \frac{\pi}{12}$, amplitude of $(z_1 z_3) = \frac{7\pi}{12}$,

and amplitude of $(z_2{}^3 z_3) = \frac{7\pi}{6}$, then the amplitude of $(z_1 z_2 z_3) =$.............

(a) $\frac{11\pi}{12}$ (b) $\frac{3\pi}{4}$ (c) $\frac{3\pi}{8}$ (d) $\frac{3\pi}{2}$

(12) $\int \frac{dx}{e^{-x}+1}$ =................

(a) $\ln(e^{-x} + 1) + c$

(b) x + c

(c) $\ln(e^{x} + 1)$+C

(d) $-\ln(e^{-x} + 1)$ +c

(13) If the measure of the angle between two planes : $3x - 6y + mz = 4$, $x + z = 7$ is 45^{o} , then m = ...

(a) 2 (b) 4 (c) 6 (d) 8

(14) If the triangle with vertices (4 , 5 , 2) , (1 , k , 3) ,(2 , 4 , 5) is an equilateral triangle, then k ∈ ....

(a) $\{\frac{21}{5}\}$ (b) {7} (c) {3 , 7} (d) $\{7 , -\frac{21}{5}\}$

(15) If the curve y=$(a-2x)^2$ has a critical point at x = -1 , then a = ...

(a) 2 (b) -1 (c) -2 (d) 1

(16) $\int \frac{(x+2)}{x^2+4x-5} dx =$.......................

(a) $\ln|x^2+4x-5|+c$

(b) $2\ln|x^2+4x-5|+c$

(c) $\frac{1}{2}\ln|x^2+4x-5|+c$

(d) $\frac{1}{2}\ln|x+2|+c$

(17) The equation of straight line passing through the point (2,1,-3) and intercepts a part of length 4 units from the positive part of z-axis is.............................

(a) $\vec{r}$ = (2 , 1, -3) + k (0 , 0 , 4)

(b) $\vec{r}$ = (0 , 0 , 4) + k (2 , 1 ,-3)

(c) $\vec{r}$ = (0 , 0 , 4) + k (2 , 1 ,-7)

(d) $\vec{r}$ = (2 , 1 ,- 3) + k $\vec{Z}$

(18) The area of the region bounded by the straight lines:

$y = \frac{1}{2}x + 4$, $x = -1$, $x = 3$ and the x – axis equals ………. square units

(a) 15 (b) 16 (c) 17 (d) 18

Third: essay questions "two marks for each question".

(19) If $1, \omega, \omega^2$ are cubic roots of 1 ,Find the value of the expression:

$$\left(\frac{3+5\omega}{3\omega^2+5} + \frac{7\omega^2-4}{7-4\omega}\right)^{15}$$

(20) A piece of wire in the shape of a circle of radius length 12 cm, it is wanted to divide it into two pieces to form a circle from each piece. Find the length of each piece that make the sum of areas of the two circles to be minimum.

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Pure Mathematics

1 $|z| = 2\sqrt{2}$, $\theta = \frac{-\pi}{4}$

in 4th quad. (b)

2 T_5 from end

$= {}^{20}C_{\cancel{4}} (x^2)^4 (-\frac{1}{x})^{20-4}$

$= {}^{20}C_4 \; x^{8} (-x^{-1})^{16} = {}^{20}C_4 \, x^{-8}$. (b)

3 The distance between the Point $(a, 2a, 3a)$ and xy-plane $= |3a| = 6$

$3a = 6$ or $3a = -6$

$a = 2$ (b) $a = -2$ refused

$(a > 0.)$

4 $y = n^3 - 1 \Rightarrow \frac{dy}{dn} = 3n^2$

$z = 1 - n^2 \Rightarrow \frac{dz}{dn} = -2n$ (b)

$\frac{dy}{dz} = \frac{dy/dn}{dz/dn} = \frac{3n^2}{-2n} = \frac{-3}{2} n.$

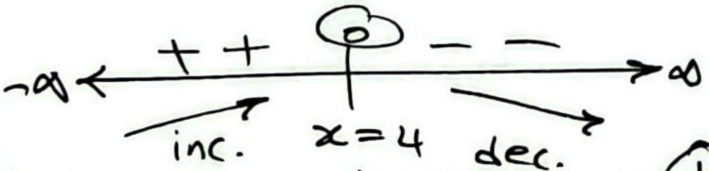
5 $f(x) = e^{8x - x^2}$

$f'(x) = (8 - 2x) e^{8x - x^2}$

Put $f'(x) = 0$: $8 - 2x = 0$

$x = 4$

and $e^{8x - x^2} = 0$ (refused)



increasing in interval $]-\infty, 4[$. (d)

6 $\vec{A} = (-2\sqrt{2}, 3, 1)$, $\|\vec{A}\| = 3\sqrt{2}$

$\because \cos\theta_x = \frac{A_x}{\|\vec{A}\|} = \frac{-2\sqrt{2}}{3\sqrt{2}} = \frac{-2}{3}$

$\therefore \theta_x \simeq 131° \, 49'$. (a)

7 $T_{r+1} = {}^{12}C_r (6x^{-2})^r (7x)^{12-r}$

$= {}^{12}C_r (6)^r (7)^{12-r} x^{-2r+12-r}$

$\therefore 12 - 3r = 0 \Rightarrow r = 4$ (a)

$\therefore$ the term free of x is T_5.

8 $y = e^x + x$ at Point $(0, 1)$

$\frac{dy}{dx} = e^x + 1$

Slope of the tangent $= \left[\frac{dy}{dx}\right]_{\text{at } x=0} = e^0 + 1 = 2.$ (C)

9 $\because x + y = 10 \Rightarrow y = 10 - x$

$\text{Area} = \frac{1}{2} bh = \frac{1}{2} xy$

$A(x) = \frac{1}{2} x (10 - x)$

$= 5x - \frac{1}{2} x^2$

$\frac{dA}{dx} = 5 - x \Rightarrow$ Put $\frac{dA}{dx} = 0$

$\frac{d^2A}{dx^2} = -1$ (-ve) $\therefore 5 - x = 0$

$x = 5$

$A_{max}(5) = 5(5) - \frac{1}{2}(5)^2 = 12.5 \text{ cm}^2$ (b)

10 $\int \frac{7x^2}{5 - 4x^3} dx = 7 \int \frac{x^2}{5 - 4x^3} dx$

$= \frac{7}{-12} \int \frac{-12x^2}{5 - 4x^3} dx$

$= -\frac{7}{12} \ln |5 - 4x^3| + C$ (C)

$\boxed{11}$ $Amp\left(\frac{z_1}{z_2}\right) = \theta_1 - \theta_2 = \frac{\pi}{12}$

$Amp(z_1 z_3) = \theta_1 + \theta_3 = \frac{7\pi}{12}$

$Amp(z_2^3 z_3) = 3\theta_2 + \theta_3 = \frac{7\pi}{6}$

By adding $\Rightarrow 2\theta_1 + 2\theta_2 + 2\theta_3 = \frac{11\pi}{6}$

$(\div 2)$

$\therefore \theta_1 + \theta_2 + \theta_3 = \frac{11\pi}{12}$

$\therefore Amp(z_1 z_2 z_3) = \frac{11\pi}{12}$. (a)

$\boxed{12}$ $\int \frac{dx}{e^{-x}+1} = \int \frac{1}{e^{-x}+1} \times \frac{e^x}{e^x}\, dx$

$= \int \frac{e^x}{1+e^x}\, dx$ ← $f'(x)$, ← $f(x)$

$= \ln|1+e^x| + C$. (c)

$\boxed{13}$ $\vec{n}_1 = (3, -6, m)$ and

$\vec{n}_2 = (1, 0, 1)$

$\cos 45° = \frac{|(3,-6,m)\cdot(1,0,1)|}{\sqrt{9+36+m^2}\sqrt{1+1}}$

$\frac{1}{\cancel{\sqrt{2}}} = \frac{|3+m|}{\sqrt{45+m^2}\cancel{\sqrt{2}}}$

$\therefore |3+m| = \sqrt{45+m^2}$ "Squaring"

$9 + 6m + m^2 = 45 + m^2$

$6m = 36 \Rightarrow m = 6$. (c)

$\boxed{14}$ $\because$ the triangle ABC is an equilat.

$\therefore$ all sides are equal

then $AB = BC$

$\sqrt{(4-1)^2+(5-k)^2+(2-3)^2} = \sqrt{(1-2)^2+(k-4)^2+(3-5)^2}$

$\sqrt{9+(5-k)^2+1} = \sqrt{1+(k-4)^2+4}$

$\sqrt{10+(5-k)^2} = \sqrt{5+(k-4)^2}$

$\therefore 10 + (5-k)^2 = 5 + (k-4)^2$

$5 + 25 - 10k + k^2$

$= k^2 - 8k + 16$

$\therefore k = 7$. (b)

$\boxed{15}$ $y = (a-2x)^2$

$\frac{dy}{dx} = 2(a-2x)(-2)$

$= -4(a-2x)$

$\because f(x)$ has a critical Point at

$x = -1$

$\left.\frac{dy}{dx}\right)_{\text{at } x=-1} = 0 \Rightarrow -4(a+2) = 0$

$\therefore a = -2$. (c)

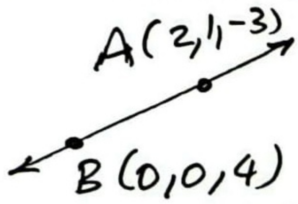
$\boxed{16}$ $\int \frac{x+2}{x^2+4x-5}\, dx$

$= \frac{1}{2}\int \frac{2x+4}{x^2+4x-5}\, dx$ ← $f'(x)$, ← $f(x)$

$= \frac{1}{2}\ln|x^2+4x-5| + C$.

(c)

$\boxed{17}$



$\vec{d} = \overrightarrow{AB}$

$= \vec{B} - \vec{A}$

$= (-2,-1,7)$ or $= (2,1,-7)$

$\therefore$ The equation of the straight line is $\vec{r} = (0,0,4) + t(2,1,-7)$.

(c)

$\boxed{18}$ $y = \frac{1}{2}x + 4$

$$\text{Area} = \int_{-1}^{3} y \, dx$$

$$= \int_{-1}^{3} \left(\frac{1}{2}x + 4\right) dx$$

$$= 18 \text{ square unit. } (d)$$

$\boxed{19}$

$$\left(\frac{3+5\omega}{3\omega^2+5} + \frac{7\omega^2-4}{7-4\omega}\right)^{15}$$

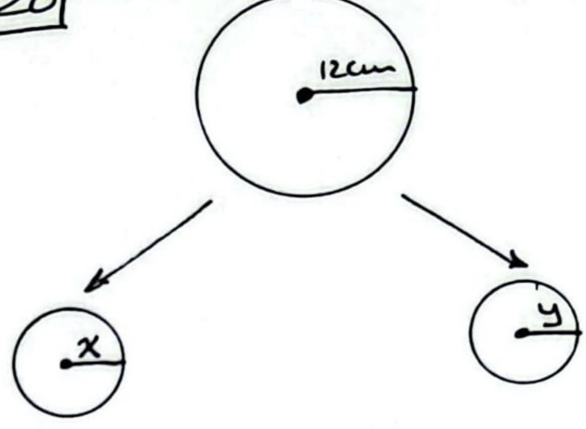
$$= \left(\frac{3\omega^3+5\omega}{3\omega^2+5} + \frac{7\omega^2-4\omega^3}{7-4\omega}\right)^{15}$$

$$= \left(\frac{\omega\cancel{(3\omega^2+5)}}{\cancel{3\omega^2+5}} + \frac{\omega^2\cancel{(7-4\omega)}}{\cancel{7-4\omega}}\right)^{15}$$

$$= (\omega + \omega^2)^{15} = (-1)^{15}$$

$$= \underline{1}.$$

$\boxed{20}$



$\therefore 2\pi(12) = 2\pi x + 2\pi y$

$\therefore x + y = 12 \Rightarrow y = 12 - x$

Sum of two Areas $= A_1 + A_2$

$= \pi x^2 + \pi y^2$

$= \pi\left(x^2 + (12 - x)^2\right)$

$= \pi\left(x^2 + 144 - 24x + x^2\right)$

$S(x) = \pi\left(2x^2 - 24x + 144\right)$

$\frac{dS}{dx} = \pi(4x - 24)$

$\frac{d^2S}{dx^2} = \pi(4) = 4\pi > 0$

Put $\frac{dS}{dx} = 0$: $\pi(4x - 24)$

$\therefore x = 6$ cm.

then the sum of two areas has a max. Value at $x = 6$cm.

$\therefore$ The length of each Piece.

$x = 6$cm. and $y = 6$cm.